13.

$$\int e^{2x}\cos(3x)dx = e^{2x}\cdot\frac{1}{3}\sin(3x) - \int\frac{1}{3}\sin(3x)\cdot 2e^{2x}dx = \frac{1}{3}e^{2x}\sin(3x) - \frac{2}{3}\int e^{2x}\sin(3x)dx =$$

$u = e^{2x} \Rightarrow du = 2e^{2x}dx$

$dv = \cos(3x)dx \Rightarrow v = \int\cos(3x)dx = \frac{1}{3}\sin(3x)$

$$\frac{1}{3}e^{2x}\sin(3x) - \frac{2}{3}\left[e^{2x}\cdot -\frac{1}{3}\cos(3x) - \int -\frac{1}{3}\cos(3x)\cdot 2e^{2x}dx\right] = \frac{1}{3}e^{2x}\sin(3x) + \frac{2}{9}e^{2x}\cos(3x) - \frac{4}{9}\int e^{2x}\cos(3x)dx$$

$u = e^{2x} \Rightarrow du = 2e^{2x}dx$

$dv = \sin(3x)dx \Rightarrow v = \int\sin(3x)dx = -\frac{1}{3}\cos(3x)$

A loop exists - original question in calculated answer $\therefore$

$$\int e^{2x}\cos(3x)dx = \frac{1}{3}e^{2x}\sin(3x) + \frac{2}{9}e^{2x}\cos(3x) - \frac{4}{9}\int e^{2x}\cos(3x)dx$$

$$\frac{13}{9}\int e^{2x}\cos(3x)dx = \frac{1}{3}e^{2x}\sin(3x) + \frac{2}{9}e^{2x}\cos(3x) \Rightarrow \int e^{2x}\cos(3x)dx = \frac{9}{13}\left(\frac{1}{3}e^{2x}\sin(3x) + \frac{2}{9}e^{2x}\cos(3x)\right)$$

14.

$$\int\frac{x^3}{\sqrt{5+3x^2}}dx = \int x^2\cdot x\left(5+3x^2\right)^{-\frac{1}{2}}dx = x^2\cdot\frac{1}{3}\left(5+3x^2\right)^{\frac{1}{2}} - \int\frac{1}{3}\left(5+3x^2\right)^{\frac{1}{2}}\cdot 2xdx = \frac{1}{3}x^2\left(5+3x^2\right)^{\frac{1}{2}} - \frac{2}{3}\int x\left(5+3x^2\right)^{\frac{1}{2}}dx$$

$u = x^2 \Rightarrow du = 2xdx$

$$dv = x\left(5+3x^2\right)^{-\frac{1}{2}}dx \Rightarrow v = \int x\left(5+3x^2\right)^{-\frac{1}{2}}dx = \frac{1}{6}\int z^{-\frac{1}{2}}dz = \frac{1}{6}\frac{z^{\frac{1}{2}}}{\frac{1}{2}} = \frac{1}{3}z^{\frac{1}{2}} = \frac{1}{3}\left(5+3x^2\right)^{\frac{1}{2}}$$

$$z = \left(5+3x^2\right) \Rightarrow dz = 6xdx \Rightarrow \frac{1}{6}dz = xdx$$

$$\frac{1}{3}x^2\left(5+3x^2\right)^{\frac{1}{2}} - \frac{2}{3}\int z^{\frac{1}{2}}dz = \frac{1}{3}x^2\left(5+3x^2\right)^{\frac{1}{2}} - \frac{2}{3}\cdot\frac{z^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{3}x^2\left(5+3x^2\right)^{\frac{1}{2}} - \frac{4}{9}z^{\frac{3}{2}} = \frac{1}{3}x^2\left(5+3x^2\right)^{\frac{1}{2}} - \frac{4}{9}\left(5+3x^2\right)^{\frac{3}{2}}$$

$$z = \left(5+3x^2\right) \Rightarrow dz = 6xdx \Rightarrow \frac{1}{6}dz = xdx$$

15.

$$\int \ln(x^2+4)dx = x\ln(x^2+4) - \int x \cdot \frac{1}{x^2+4} \cdot 2xdx = x\ln(x^2+4) - 2\int \frac{x^2}{x^2+4}dx = x\ln(x^2+4) - 2\int 1 - \frac{4}{x^2+4}dx$$

$$u = \ln(x^2+4) \Rightarrow du = \frac{1}{x^2+4} \cdot 2xdx$$

$$dv = dx \Rightarrow v = \int dx \Rightarrow v = x$$

$$x\ln(x^2+4) - 2\left[\int 1dx - 4\int \frac{1}{x^2+4}dx\right] = x\ln(x^2+4) - 2x + \frac{8}{2}\tan^{-1}\frac{x}{2} = x\ln(x^2+4) - 2x + 4\tan^{-1}\frac{x}{2}$$

16.

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x = e^x(x-1)$$

$$u = x \Rightarrow du = dx$$

$$dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

17.

$$\int x\sin(4x)dx = \frac{1}{4}x\cos(4x) - \int -\frac{1}{4}\cos(4x)dx = \frac{1}{4}x\cos(4x) + \frac{1}{4}\int \cos(4x)dx = \frac{1}{4}x\cos(4x) + \frac{1}{4}\left(\frac{1}{4}\sin(4x)\right) =$$

$$u = x \Rightarrow du = dx$$

$$dv = \sin(4x)dx \Rightarrow v = \int \sin(4x)dx = \frac{1}{4} \cdot -cos(4x) = -\frac{1}{4}\cos(4x)$$

$$\frac{1}{4}x\cos(4x) + \frac{1}{16}\sin(4x)$$

18.

$$\int x^2\sin(2x)dx = -\frac{1}{2}x^2\cos(2x) - \int -\frac{1}{2}\cos(2x)2xdx = -\frac{1}{2}x^2\cos(2x) + \int x\cos(2x)dx =$$

$$u = x^2 \Rightarrow du = 2xdx$$

$$dv = \sin(2x)dx \Rightarrow v = \int \sin(2x)dx = -\frac{1}{2}\cos(2x)$$

$$-\frac{1}{2}x^2\cos(2x) + \left[\frac{1}{2}x\sin(2x) - \int \frac{1}{2}\sin(2x)dx\right] = -\frac{1}{2}x^2\cos(2x) + \left[\frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)dx\right] =$$

$$u = x \Rightarrow du = dx$$

$$dv = \cos(2x)dx \Rightarrow v = \int \cos(2x)dx = \frac{1}{2}\sin(2x)$$

$$-\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) - \frac{1}{2}\left(-\frac{1}{2}\cos(2x)\right) = -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)$$